Sample Paper – 2
Mathematics
Class XI Session 2022-23

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A

(Multiple Choice Questions)
Each question carries 1 mark.

1. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then:
(a) $x = 2n, n \in Z$ (b) $x = 2n + 1, n \in N$
(c) $x = 4n, n \in Z$ (d) $x = 4n + 1, n \in N$ 1

2. Meenakshi thought for two different finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively:
(a) 4, 7 (b) 7, 4
(c) 4, 4 (d) 7, 7 1

3. Equation of the line passing through (1, 2) and parallel to the line $y = 3x - 1$ is:
(a) $y + 2 = x + 1$ (b) $y + 2 = 3(x + 1)$
(c) $y - 2 = 3(x - 1)$ (d) $y - 2 = x - 1$ 1

4. If $f(x) = \frac{|x-9|}{x-9}$, then find $\lim_{x \to 3^3} f(x)$.
(a) 1 (b) –1
(c) 2 (d) 0 1

5. If x is a real number and $|x| < 3$, then x lies between:
(a) $x \geq 3$ (b) $x \leq -3$
(c) $-3 \leq x \leq 3$ (d) $-3 < x < 3$ 1

6. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is:
(a) 216 (b) 600
(c) 240 (d) 3125 1

7. The mean deviation about the median of the following data is:
21, 30, 24, 32, 31
(a) 5.28 (b) 5.18
(c) 3.38 (d) 3.6 1

8. If the parabola passes through the point (–3, –2), then the length of its latus rectum is, if equation of parabola is $y^2 = 4ax$:
(a) $-\frac{2}{3}$ (b) $-\frac{4}{3}$
(c) $-\frac{1}{3}$ (d) – 4 1

9. If $p = e^{2x-2}$, then log p at $x = 3$ is:
(a) 1 (b) 2
(c) 3 (d) 4 1

10. If $2x - 1 < 6 + x, 4 - 3x \leq 1$, then $x \in$:
(a) [1, 7] (b) [–1, 7]
(c) [1, 7) (d) (1, 7) 1

11. The mean deviation from the mean of the first three natural numbers is:
(a) 0.667 (b) 0.253
(c) 0.456 (d) None 1

12. A couple has two children, the probability that both children are males, if it is known that at least one of the children is male is:
(a) 2/3 (b) 1/3
(c) 4/5 (d) 5/3 1

13. A teacher write an expression on board. Then she asked the students what is the number of terms in the expansion of $(1 + q)^9 + (1 - q)^9$. The number of terms in the expansion is:

(a) 5 (b) 7

(c) 9 (d) 10 1

14. A box contains 1 red and 3 identical blue balls. Two balls are drawn at random in succession without replacement. Then, the sample space for this experiment is:

(a) {RB,BR,BB} (b) {R, B, B}

(c) {RB} (d) {RB, BR} 1

15. The value of $\lim_{x \to 0}\left[\frac{\sqrt{3+x}-\sqrt{3}}{x}\right]$ is equal to:

(a) $\frac{1}{3(2)^{\frac{1}{2}}}$ (b) $\frac{1}{2(3)^{\frac{1}{2}}}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$ 1

16. Priya has a wall clock in her living hall. The large hand of the clock is 49 cm long. How much distance does its extremity move in 30 minutes?

(a) 154 cm (b) 80 cm

(c) 75 cm (d) 77 cm 1

17. If $3x - 1 < 5 + x, 6 - 5x \leq 1$, then $x \in$:

(a) [1, 3] (b) [– 1, 3]

(c) [1, 3) (d) (1, 3) 1

18. What is the perpendicular distance of the point $P(5, 6, 7)$ from xy-plane?

(a) 8 units (b) 7 units

(c) 6 units (d) 5 units 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of the reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): The sum of the first 20 terms of an A.P.: 4, 8, 12 .... is equal to 840.

Reason (R): Sum of n terms of an A.P is $S_n = \frac{n}{2}(2a + (n-1)d$. 1

20. Let α be a real number lying between 0 and $\pi/2$ and n be a positive integer.

Assertion (A): $\tan \alpha + 2 \tan 2\alpha + 2^2 \tan 2^2 \alpha + ... + 2^{n-1} \tan 2^{n-1} \alpha + 2^n \cot 2^n \alpha = \cot \alpha$

Reason (R): $\cot \alpha - \tan \alpha = 2 \cot 2\alpha$. 1

SECTION - B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. For any two sets A and B, prove that:

(A) $A \cup (B - A) = A \cup B$.

(B) $(A \cap B) \cup (A - B) = A$

OR

Given A = {1, 2, 3, 4, 5}, B = {2, 4, 6} and C = {3, 5, 7}.

Find $A - (B \cup C)$ and $A - (B \cap C)$. 2

22. For what value of 'p', the coefficients of $(2p + 1)^{th}$ and $(4p + 5)^{th}$ terms in the expansion of $(1 + a)^{10}$ are equal? 2

23. Find the point on x-axis which is equidistant from the points A(0, 3, 2) and B(5, 0, 4). 2

24. Write the sum of $2 + 4 + 6 + 8 + ... + 2n$.

OR

If $x \in R$, find the minimum value of the expression $3^x + 3^{1-x}$. 2

25. If $f(x) = x^4$, then find the range of the function for $x = \{1, 2, 3, 4, 5\}$. 2

SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find the range of each of the following functions:

(A) $f(x) = |x - 6|$ (B) $f(x) = 1 - |x - 4|$

(C) $f(x) = \frac{|x-2|}{x-2}$ 3

27. Find the derivative of $\frac{a + b \sin x}{c + d \cos x}$. 3

28. Find the equation of the line(s) which passes through the point (3, 4) and cuts off intercepts on the coordinates axes such that their sum is 14.

OR

Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, –1). 3

29. Find the variance and standard deviation for the following distribution:

x	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f	1	5	12	22	17	9	4

3

30. Evaluate: $\lim_{x \to 0}\left[\frac{\tan x - 2\tan 3x + \tan 5x}{x}\right]$.

OR

The circular measures of two angles of a triangle are 1/2 and 1/3, find the third angle in the English system. 3

31. If $2\sin^2\theta = 3\cos\theta$ where, $0 \le \theta \le 2\theta$, then find the value of θ.

OR

Find the value of

$\sin\frac{\pi}{18} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \sin\frac{5\pi}{18}$. 3

SECTION - D

(This section comprises of long answer-type questions (LA) of 5 marks each.)

32. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α, β lie between 0 and $\frac{\pi}{4}$, then find the value of $\tan 2\alpha$. 5

33. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many ways:
(A) four cards are of different colour?
(B) four cards belong to four different suits?
(C) four cards are face cards?
(D) two are red cards and two are black cards?
(E) cards are of the same colour? 5

34. Find the sum of 5 + 5.5 + 5.55 . . . to n terms.

OR

If S be the sum, P be the product and R be the sum of the reciprocals of n terms of a G.P., then prove that $\left(\frac{S}{R}\right)^n = P^2$. 5

35. Three coins are tossed once. Find the probability of getting:
(A) 3 heads
(B) 2 heads
(C) at least 2 heads
(D) at most 2 heads
(E) no head

OR

If the length of the 'ASSASSINATION' are arranged at random. Find the probability that
(A) four S's come consecutively in the word.
(B) two I's and two N's come together.
(C) all A's are not coming together.
(D) no two A's are coming together. 5

SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (A), (B), (C) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. Case-Study 1:

A complex number z is purely real if and only if $\bar{z} = z$ and is purely imaginary if and only if $\bar{z} = -z$.

(A) If $(1 + i)z = (1 - i)\bar{z}$, find the value of $-i\bar{z}$. 1
(B) Find the value of $\overline{z_1 z_2}$. 1
(C) If x and y are real numbers and the complex number

$\frac{(2+i)x - i}{4+i} + \frac{(1-i)y + 2i}{4i}$

is purely real, then what is the relation between x and y?

OR

If $z = \frac{3 + 2i\sin q}{1 - 2i\sin q}\left(0 < q \le \frac{p}{2}\right)$ is pure imaginary, find the value of θ. 2

37. Case-Study 2:

To check the understanding of sets, a Math teacher writes two sets A and B having finite *numbers of elements. The sum of cardinal numbers of two finite sets A and B is 9. The ratio of a cardinal number of the power set of A is to a cardinal number of the power set of B is 8 : 1.*

(A) What is the cardinal number of set A? 1

(B) What is the cardinal number of set B? 1

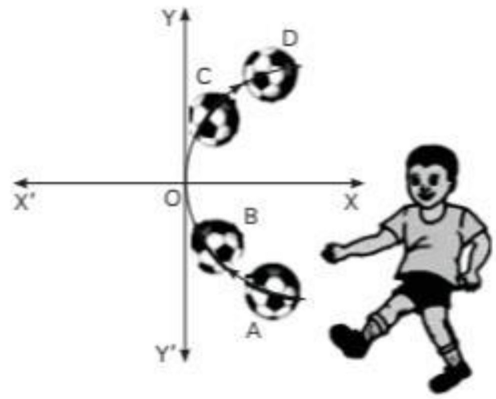
(C) Find the maximum value of $n(A \cup B)$.

OR

Find the minimum value of $n(A \cup B)$. 2

38. Case-Study 3:

Arun was playing a football match. When he kicked the football, the path formed by the football from ground level is parabolic, which is shown in the following graph. Consider the coordinates of point A as (3, –2).



(A) Find the distance between the extremities of latus rectum of the given curve. 2

(B) Find the equation of directrix of the path covered by ball. 2

SOLUTION

SECTION - A

1. (c) $x = 4n, n \in Z$

Explanation: If $\left(\frac{1+i}{1-i}\right)^x = 1$

Multiplying and dividing with $(1+i)^x$ in LHS

$$\Rightarrow \quad \left(\frac{(1+i)}{(1-i)}\right)^x = \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^x$$

$$= \left(\frac{1-1+2i}{2}\right)^x$$

$$= i^x$$

But $\quad i^x = 1 = i^4$

or $\quad i^x = i^{4n} \ (n \in Z)$

Therefore $\quad x = 4n, n \in Z$

2. (b) 7, 4

Explanation: Given that m and n are elements of two finite sets.

Number of subsets of first set $= 2^m$

Number of subsets of second set $= 2^n$

According to question,

$$2^m - 2^n = 112 = 2^4 \times 7$$

$$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$$

On comparing, we get

$$n = 4 \text{ and } m - n = 3$$

$$\Rightarrow \quad n = 4 \text{ and } m = 7$$

3. (c) $y - 2 = 3(x - 1)$

Explanation: Line is parallel to $y = 3x - 1$ and passes through the point (1, 2). Hence, its slope is same as that of the given line *i.e.,* $m = 3$

So, the equation of the line is: $y - 2 = 3(x - 1)$

4. (b) –1

Explanation:

We have, $\quad f(x) = \frac{|x-9|}{x-9}$

LHL $\quad = \frac{-(x-9)}{x-9}$

$= -1$

So, $\quad \lim_{x \to 3^-} f(x) = -1$

5. (d) $-3 < x < 3$

Explanation: $|x| < 3$

Here, x is a real number

So,

$x < 3$

$-x < 3 \Rightarrow x > -3$

$\Rightarrow \quad x \in (-3, 3)$

6. *(a) 216*

Explanation: 5-digit numbers divisible by that can be formed using digits 0, 1, 2, 4, 5

4	4	3	2	1

is $4 \times 4 \times 3 \times 2 \times 1 = 96$

5-digit numbers divisible by 3 that can be formed using digits 1, 2, 3, 4, 5 = 5!

$\therefore$ Total number of ways $= 5! + 96$

$= 120 + 96$

$= 216$

Caution

➥ *A number is divisible by 3 if the sum of its digits is divisible by 3.*

7. *(d) 3.6*

Explanation: $x_i = 21, 24, 30, 31, 32$

Number of observations $= n = 5$ (odd)

Since n is odd

$$\text{Median} = \left(\frac{5+1}{2}\right)^{th} \text{term}$$

$$= 3^{rd} \text{ term}$$

$$M = 30$$

$$M.D = \frac{9+6+0+1+2}{5}$$

$$= \frac{18}{5}$$

$$= 3.6$$

8. *(b)* $-\frac{4}{3}$

Explanation: Here equation of parabola is

$$y^2 = 4ax$$

It passes through $(-3, -2)$, so

$$4 = -12a$$

$$\Rightarrow \quad -\frac{1}{3} = a$$

Then, length of latus rectum $= 4a$

$$= 4 \times -\frac{1}{3} = -\frac{4}{3}$$

9. *(d) 4*

Explanation: $p = e^{2x-2}$

Taking log on both sides

$$\log p = \log e^{2x-2}$$

$$= (2x-2) \log e$$

$\log p = 2x - 2 \qquad (\log e = 1)$

$\log p = 2 \times 3 - 2$

$= 6 - 2$

$= 4$

10. *(c) (1, 7)*

Explanation: We have, $2x - 1 < 6 + x$

$\Rightarrow \quad x < 7$

Also, we have $4 - 3x \le 1$

$\Rightarrow \quad -3x \le -3$

$\Rightarrow \quad x \ge 1$

Thus, solution of the given system are all real numbers lying between 1 and 7 including 1, *i.e.*, $1 \le x < 7$.

$\therefore \quad x \in [1, 7)$

11. *(a) 0.667*

Explanation: First 3 natural numbers are 1, 2, 3.

Then, mean $= \frac{1+2+3}{3}$ term

$= \frac{6}{3} = 2$ term

Hence, mean $(\bar{x}) = 2$

So, $\Sigma|x_i - \bar{x}| = 1 + 0 + 1 = 2$

$$\text{M.D. } (\bar{x}) = \frac{2}{3} = 0.667$$

12. *(b) 1/3*

Explanation: S = {MM, MF, FM, FF}

Total number of possible outcomes is 4.

E : Both children are males.

F : Atleast one child is male

E = {(M, M)}

F = {(M, F) (F, M) (M, M)}

Now, $P(E) = \frac{1}{4}, P(F) = \frac{3}{4}$

and $P(E \cap F) = \frac{1}{4}$

So, required probability, $P(E|F) = \frac{P(E \cap F)}{P(F)} \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

13. *(a) 5*

Explanation: We have,

$$(1+q)^9 + (1-q)^9$$

$$= 2({}^9C_0 + {}^9C_2\,(q)^2 + \ldots + {}^9C_8\,(q)^8)$$

Thus, it has 5 terms.

14. *(a) {RB, BR, BB}*

Explanation: Let red ball is denoted by *R* and blue ball is denoted by *B*. Now, two balls drawn at random in succession without replacement.

Then, the sample space is S = {RB, BR, BB}

15. *(b)* $\frac{1}{2(3)^{\frac{1}{2}}}$

Explanation: We have, $\lim_{x\to 0}\left[\frac{\sqrt{3+x}-\sqrt{3}}{x}\right]$

We can write as $\frac{(3+x)^{1/2}-(3)^{1/2}}{(x+3)-(3)}$

Now, using $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

$$\text{Given limit} = \frac{1}{2}(3)^{1/2-1}$$

$$= \frac{1}{2}(3)^{\frac{-1}{2}}$$

$$= \frac{1}{2(3)^{\frac{1}{2}}}$$

16. *(a) 154 cm*

Explanation: The large hand of the clock makes a complete revolution in 60 minutes.

∴ Angle traced out by the large hand in 30 minutes (of time) = $\frac{360°\times 30}{60°}$ = 180° = $\frac{180°}{180}\pi$ radian = π radian

Hence, the distance moved by the extremity of the large hand = 49 × π = 49 × 22/7 = 154 cm.

17. *(c) [1, 3)*

Explanation: We have, $3x-1<5+x \Rightarrow x<3$

Also, we have $6-5x\le 1 \Rightarrow -5x \le -5 \Rightarrow x \ge 1$

Thus, solution of the given system are all real number lying between 1 and 3 including 1, *i.e.,*

$$1 \le x < 3.$$

$$\therefore \quad x \in [1, 3)$$

18. *(b) 7 units*

Explanation: Let *L* be the foot of perpendicular drawn from the point *P*(5, 6, 7) on the *xy*-plane, *i.e.,* $z = 0$

Then, coordinates of *L* = (5, 6, 0)

∴Required distance, *PL*

$$= \sqrt{(5-5)^2+(6-6)^2+(7-0)^2} = 7 \text{ units.}$$

19. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Given A.P. is 4, 8, 12 ....

So, $a = 4, d = 4$

Now, $S_{20} = \frac{20}{2}(2\times 4+(20-1)\times 4) = 840$

20. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Now,

$$\cot\alpha - \tan\alpha = \frac{1}{\tan\alpha} - \tan\alpha = \frac{1-\tan^2\alpha}{\tan\alpha}$$

$$= 2\left(\frac{1-\tan^2\alpha}{2\tan\alpha}\right) = 2\cot 2\alpha$$

From here, we get $\tan\alpha = \cot\alpha - 2\cot 2\alpha$

Making repeated use of this identify, we shall obtain

$\tan\alpha + 2\tan 2\alpha + 2^2\tan 2^2\alpha + ... + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha$

$= (\cot\alpha - 2\cot 2\alpha) + 2(\cot 2\alpha - 2\cot 2^2\alpha) + 2^2(\cot 2^2\alpha - 2\cot 2^3\alpha) + ... + 2^{n-1}(\cot 2^{n-1}\alpha - 2\cot 2^n\alpha) + 2^n\cot 2^n\alpha = \cot\alpha$

SECTION - B

21. (A) $A \cup (B - A) = A \cup (B \cap A')$

$= (A \cup B) \cap (A \cup A')$ [By Distributive Law]

$= (A \cup B) \cap U = A \cup B.$

(B) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$

$= A \cap (B \cup B')$ [By Distributive Law]

$= A \cap U = A.$

⚠ **Caution**

- *The difference between A-B and B-A is that first means only A and second means only B.*

OR

Given

A = {1, 2, 3, 4, 5}, B = {2, 4, 6} and C = {3, 5, 7}.

Then,

$B \cup C = \{2, 4, 6\} \cup \{3, 5, 7\} = \{2, 3, 4, 5, 6, 7\}$

$B \cap C = \{2, 4, 6\} \cap \{3, 5, 7\} = \phi$

Hence,

$A - (B \cup C) = \{1, 2, 3, 4, 5\} - \{2, 3, 4, 5, 6, 7\}$
$= \{1, 6, 7\}$.

And

$A - (B \cap C) = \{1, 2, 3, 4, 5\} - \phi = \{1, 2, 3, 4, 5\}$

22. Coeff. of $(2p + 1)^{th}$ term in $(1 + a)^{10} = {}^{10}C_{2p}$

and coeff. of $(4p + 5)^{th}$ term in $(1 + a)^{10}$

$= {}^{10}C_{4p+4}$.

By the question, ${}^{10}C_{2p} = {}^{10}C_{4p+4}$

Thus, either $10 = 2p + (4p + 4)$ or $2p = 4p + 4$

$\Rightarrow 6p = 6$ or $2p = -4$

$\Rightarrow p = 1$ or $p = -2$.

But p can't be –ve.

Hence, $p = 1$.

23. Let P(x, 0, 0) be the point on x-axis which is equidistant from the point A(0, 3, 2) and B(5, 0, 4).

$\therefore$ AP = BP

$$\Rightarrow \sqrt{(x-0)^2+(0-3)^2+(0-2)^2} = \sqrt{(x-5)^2+(0-0)^2+(0-4)^2}$$

On squaring both sides, we get

$$(x-0)^2 + 9 + 4 = (x-5)^2 + 0 + 16$$

$$\Rightarrow x^2 + 13 = x^2 + 25 - 10x + 16$$

$$\Rightarrow 10x = 28$$

$$\Rightarrow x = \frac{14}{5}$$

Hence, the required point is $\left(\frac{14}{5}, 0, 0\right)$.

24. Here, n^{th} term is $2n$

So, $S_n = 2 + 4 + 6 + 8 + \dots 2n$

$$= \frac{n}{2}[4+(n-1)2]$$

$$= \frac{n}{2}[2+2n]$$

$$= n(n+1)$$

$$= n^2 + n$$

OR

We know that A.M. $\geq$ G.M.

$$\therefore \frac{3^x + 3^{1-x}}{2} \geq \sqrt{3^x \times 3^{1-x}} \text{ for all } x \in R$$

$$\Rightarrow \frac{3^x + 3^{1-x}}{2} \geq \sqrt{3} \text{ for all } x \in R$$

$$\Rightarrow 3^x + 3^{1-x} \geq 2\sqrt{3} \text{ for all } x \in R$$

Hence, the minimum value of $3^x + 3^{1-x}$ for any $x \in R$ is $2\sqrt{3}$

25. Here, $f(x) = x^4$

For $x = 1$

$f(x) = 1^4 = 1$

For $x = 2$

$f(x) = 2^4 = 16$

For $x = 3$

$f(x) = 3^4 = 81$

For $x = 4$

$f(x) = 4^4 = 256$

For $x = 5$

$f(x) = 5^4 = 625$

Hence, range of $f(x) = x^4$ for $x = \{1, 2, 3, 4, 5\}$ is $= \{1, 16, 81, 256, 625\}$

SECTION - C

26. (A) We have $f(x) = |x - 6|$

Clearly, $f(x)$ is defined for all $x \in R$.

Therefore, Domain $(f) = R$.

$\because |x - 6| > 0$ for all $x \in R$

$\therefore 0 \leq |x - 6| < \infty$ for all $x \in R$

$\Rightarrow 0 \leq f(x) < \infty$ for all $x \in R$

$\Rightarrow f(x) \in [0, \infty)$ for all $x \in R$

Hence, Range $(f) = [0, \infty)$.

(B) We have, $f(x) = 1 - |x - 4|$.

We observe that $f(x)$ is defined for all $x \in R$.

Therefore, Domain $(f) = R$.

$\because 0 \leq |x - 4| < \infty$ for all $x \in R$

$\therefore -\infty < -|x - 4| \leq 0$ for all $x \in R$

$\Rightarrow -\infty < 1 - |x - 4| \leq 1$ for all $x \in R$

$\Rightarrow -\infty < f(x) \leq 1$ for all $x \in R$

$\Rightarrow f(x) \in (-\infty, 1]$

Hence, Range $(f) = (-\infty, 1]$

(C) We have $f(x) = \frac{|x-2|}{x-2}$

Clearly, $f(x)$ is defined for all $x \in R$ except at $x = 2$.

Therefore, Domain $(f) = R - \{2\}$

Now,

$$f(x) = \frac{|x-2|}{x-2}$$

Hence, Range $(f) = \{-1, 1\}$

27. Given, $y=\frac{a+b\sin x}{c+d\cos x}$

Applying quotient rule of differentiation that is

$$\Rightarrow \frac{d}{dx}\left(\frac{t}{s}\right)=\frac{s\cdot\frac{dt}{dx}-t\cdot\frac{ds}{dx}}{s^2}$$

$$\Rightarrow \quad y=\frac{a+b\sin x}{c+d\cos x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(c+d\cos x)\frac{d}{dx}(a+b\sin x)-(a+b\sin x)\frac{d}{dx}(c+d\cos x)}{(c+d\cos x)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(c+d\cos x)(b\cos x)-(a+b\sin x)(-d\sin x)}{(c+d\cos x)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{b\cos x(c+d\cos x)+d\sin x(a+b\sin x)}{(c+d\cos x)^2}$$

$$=\frac{[cb\cos x+bd\cos^2 x+ad\sin x+bd\sin^2 x]}{(c+d\cos x)^2}$$

$$=\frac{[cb\cos x+ad\sin x+bd\,(\cos^2 x+\sin^2 x)]}{(c+d\cos x)^2}$$

$$=\frac{cb\cos x+ad\sin x+bd}{(c+d\cos x)^2}$$

28. The equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \quad ...(1)$$

Where a and b are the intercepts on the axis.

Given that $a+b=14$

$$\Rightarrow \quad b=14-a$$

Substituting the value of a and b in equation (1) we get

$$\frac{x}{a}+\frac{y}{14-a}=1$$

Taking LCM

$$\Rightarrow \quad \frac{x(14-a)+ay}{(a)(14-a)}=1$$

$$\Rightarrow \quad 14x-ax+ay=14a-a^2 \quad ...(2)$$

If equation 2 passes through the point (3, 4), then

$$14(3)-a\,(3)+a\,(4)=14a-a^2$$

$$\Rightarrow \quad 42-3a+4a-14a+a^2=0$$

$$\Rightarrow \quad a^2-13a+42=0$$

$$\Rightarrow \quad a^2-7a-6a+42=0$$

$$\Rightarrow \quad a(a-7)-6(a-7)=0$$

$$\Rightarrow \quad (a-6)\,(a-7)=0$$

$$\Rightarrow \quad a-6=0 \text{ or } a-7=0$$

$$\Rightarrow \quad a=6 \text{ or } a=7$$

If $a = 6$, then

$$6+b=14$$

$$\Rightarrow \quad b=14-6=8$$

If $a = 7$, then

$$7+b=14$$

$$\Rightarrow \quad b=14-7$$

$$\Rightarrow \quad =7$$

If $a = 6$ and $b = 8$, then equation of line is

$$\frac{x}{6}+\frac{y}{8}=1$$

$$\Rightarrow \quad \frac{4x+3y}{24}=1$$

$$\Rightarrow \quad 4x+3y=24$$

If $a = 7$ and $b = 7$, then equation of line is

$$\frac{x}{7}+\frac{y}{7}=1$$

$$\Rightarrow \quad x+y=7$$

OR

Given points are A(5 ,2), B(2, 3) and C(3, –1).

Firstly, we find the slope of the line joining the points B and C.

Slope of the line joining two points = $\frac{y_2-y_1}{x_2-x_1}$

$$\therefore \quad m_{BC}=\frac{-1-3}{3-2}=-\frac{4}{1}=-4$$

It is given that line passing through the point (5, 2) is perpendicular to BC

$$\therefore \quad m_1\,m_2=-1$$

$$\Rightarrow \quad -4\times m_2=-1$$

$$\Rightarrow \quad m_2=\frac{1}{4}$$

Therefore, slope of the required line = $\frac{1}{4}$

Now, we have to find the equation of line passing though point (5, 2)

Equation of line : $y-y_1=m(x-x_1)$

$$\Rightarrow \quad y-2=\frac{1}{4}(x-5)$$

$$\Rightarrow \quad 4y-8=x-5$$

$$\Rightarrow \quad x-5-4y+8=0$$

$$\Rightarrow \quad x-4y+3=0$$

Hence, the equation of line passing through the point (5, 2) is $x-4y+3=0$.

29. Calculation of variance and standard deviation

x_i	f_i	$d_i = x_i - 34.5$	$u_i = \frac{x_i - 34.5}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
4.5	1	-30	-3	-3	9	9
14.5	5	-20	-2	-10	4	20
24.5	12	-10	-1	-12	1	12
34.5	22	0	0	0	0	0
44.5	17	10	1	17	1	17
54.5	9	20	2	18	4	36
64.5	4	30	3	12	9	36
	$N = \Sigma f_i = 70$			$\Sigma f_i u_i = 22$		$\Sigma f_i u_i^2 = 130$

Here, $N = 70, \Sigma f_i u_i = 22, \Sigma f_i u_i^2 = 130$ and $h = 10$

$$\therefore \quad Var(X) = h^2 [(\frac{1}{N}\sum f_i u_i^2) - (\frac{1}{N}\sum f_i u_i)^2]$$

$$\Rightarrow \quad Var(X) = 100\left[\frac{130}{70} - \left(\frac{22}{70}\right)^2\right]$$

$$= 100\left[\frac{13}{7} - \left(\frac{11}{35}\right)^2\right]$$

$$= 100\,[1.857 - 0.098] = 175.9$$

Hence, $\quad S.D. = \sqrt{Var(X)} = \sqrt{175.9} = 13.262$

30. $\lim_{x\to 0}\left[\frac{\tan x - 2\tan 3x + \tan 5x}{x}\right]$

$$= \lim_{x\to 0}\left[\frac{\tan x}{x} - \frac{2\tan 3x}{x} + \frac{\tan 5x}{x}\right]$$

$$= \lim_{x\to 0}\left[\frac{\tan x}{x} - 6\frac{\tan 3x}{3x} + 5\frac{\tan 5x}{5x}\right]$$

$$= \lim_{x\to 0}\left[\frac{\tan x}{x}\right] - 6\lim_{x\to 0}\left[\frac{\tan 3x}{3x}\right] + 5\lim_{x\to 0}\left[\frac{\tan 5x}{5x}\right]$$

$$= \lim_{x\to 0}\left[\frac{\tan x}{x}\right] - 6\lim_{3x\to 0}\left[\frac{\tan 3x}{3x}\right] + 5\lim_{5x\to 0}\left[\frac{\tan 5x}{5x}\right]$$

$$= 1 - 6(1) + 5(1) = 0$$

OR

We know that the sum of three angles of a triangle is 180°, *i.e.*, π radian

$\therefore$ The third angle $= \left(\pi - \frac{1}{2} - \frac{1}{3}\right)$ radian

$$= \left(\frac{22}{7} - \frac{1}{2} - \frac{1}{3}\right)^C = \left(\frac{97}{42} \times \frac{180}{\pi}\right)^\circ \quad (\because \pi^C = 180^\circ)$$

$$= \left(\frac{97 \times 30}{22}\right)^\circ = \frac{1455}{11} \text{ degree}$$

$$= \left(132\frac{3}{11}\right)^\circ = 132^\circ + \left(\frac{3\times 60}{11}\right)' \quad (\because 1^\circ = 60')$$

$$= 132^\circ + \left(16\frac{4}{11}\right)' = 132^\circ + 16' + \left(\frac{4}{11}\times 60\right)''$$

$(\because 1' = 60'')$

$= 132^\circ\, 16'\, 22''$

31. Given, $2\sin^2\theta = 3\cos\theta$

$\Rightarrow \; 2(1 - \cos^2\theta) = 3\cos\theta$

$\Rightarrow \; 2 - 2\cos^2\theta - 3\cos\theta = 0$

$\Rightarrow \; 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow \; 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow \; 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$(\cos\theta + 2)(2\cos\theta - 1) = 0$

So, either $\cos\theta + 2 = 0$ or $2\cos\theta - 1 = 0$

But, $\cos\theta \neq -2 \quad [-1 \le \cos\theta \le 1]$

So, $2\cos\theta - 1 = 0$

$$\Rightarrow \; \cos\theta = \frac{1}{2}$$

$$\Rightarrow \; \cos\theta = \cos\frac{\pi}{3} \text{ or } \cos\theta = \cos\left(2\pi - \frac{\pi}{3}\right)$$

$$\Rightarrow \; \cos\theta = \cos\frac{\pi}{3} \text{ or } \cos\theta = \cos\frac{5\pi}{3}$$

OR

Given, $\sin\frac{\pi}{18} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \sin\frac{5\pi}{18}$

$= \sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ$

$= \sin 50^\circ + \sin 10^\circ + \sin 40^\circ + \sin 20^\circ$

$= \sin(180^\circ - 130^\circ) + \sin 10^\circ + \sin(180^\circ - 140^\circ) + \sin 20^\circ$

$= \sin 130^\circ + \sin 10^\circ + \sin 140^\circ + \sin 20^\circ$

$[\therefore \sin(\pi - \theta) = \sin\theta]$

$= 2\sin 70^\circ \cos 60^\circ + 2\sin 80^\circ \cos 60^\circ$

$\left[\therefore \sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$

$$= 2.\frac{1}{2}\sin 70^\circ + 2.\frac{1}{2}\sin 80^\circ$$

$$= \sin 70^\circ + \sin 80^\circ = \sin\frac{7\pi}{18} + \sin\frac{4\pi}{9}$$

SECTION - D

32. Given, $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$

$\Rightarrow \quad \sin(\alpha+\beta)=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\pm\frac{3}{5}$

$\therefore \quad \sin(\alpha+\beta)=\frac{3}{5}$

And $\quad \cos(\alpha-\beta)=\sqrt{1-\frac{25}{169}}=\sqrt{\frac{144}{169}}=\pm\frac{12}{13}$

$\therefore \quad \cos(\alpha-\beta)=\frac{12}{13}$

Now, $\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \quad \left[\because \quad 0<\alpha<\frac{\pi}{4}\right]$

$=\frac{\frac{3}{5}}{\frac{4}{5}}=\frac{3}{4}$

And $\quad \tan(\alpha-\beta)=\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\frac{5}{13}}{\frac{12}{13}}=\frac{5}{12}$

$\therefore \quad \tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta)$

$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}=\frac{\frac{(36+20)}{48}}{\frac{(48-15)}{48}}$

$=\frac{56}{33}$

33. Four cards can be chosen from 52 playing cards in ${}^{52}C_4$ ways.

Now, ${}^{52}C_4=\frac{52!}{48!\,4!}=\frac{49\times50\times51\times52}{2\times3\times4}=270725$

Hence, required number of ways = 270725

(A) There are four suits (diamond, spade, club and heart) of 13 cards each. Therefore, there are ${}^{13}C_4$ ways of choosing 4 diamond cards, ${}^{13}C_4$ ways of choosing 4 club cards, ${}^{13}C_4$ ways of choosing 4 spade cards and ${}^{13}C_4$ ways of choosing heart cards.

$\therefore$ Required number of ways

$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times \frac{13!}{9!4!}$

$= 2860$

(B) There are 13 cards in each suit. Four cards drawn belong to four different suits means one card is drawn from each suit. Out of 13 diamond cards one card can be drawn in ${}^{13}C_1$ ways.

Similarly, there are ${}^{13}C_1$ ways of choosing one club card, ${}^{13}C_1$ ways of choosing one spade card and ${}^{13}C_1$ ways of choosing one heart card.

$\therefore$ Number of ways of selecting one card from each suit $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$= 4 \times {}^{13}C_1$

$= 4 \times \frac{13!}{1!12!}$

$= 4 \times 13$

$= 52$

(C) There are 12 face cards out of which 4 cards can be chosen in ${}^{12}C_4$ ways.

$\therefore$ Required number of ways

$= {}^{12}C_4$

$= \frac{12!}{4!8!}$

$= 495$

(D) There are 26 red cards and 26 black cards. Therefore, 2 red cards can be chosen in ${}^{26}C_2$ ways and 2 black cards can be chosen in ${}^{26}C_2$ ways. Hence, 2 red and 2 black cards can be chosen in ${}^{26}C_2 \times {}^{26}C_2$

$= \left(\frac{26!}{24!2!}\right)^2$

$= (325)^2$

$= 105625$

(E) Out of 26 red cards, 4 red cards can be chosen in ${}^{26}C_4$ ways. Similarly, 4 black cards can be chosen in ${}^{26}C_4$ ways.

Hence, 4 red or 4 black cards can be chosen in ${}^{26}C_4 + {}^{26}C_4$ ways.

$= 2\times\frac{26!}{4!22!}$

$= 29900$

34. Let S be the sum of the series 5 + 5.5 + 5.55 + 5.555 + . . . to *n* terms.

S = 5 + 5.5 + 5.55 + 5.555 + . . . to *n* terms.

$\Rightarrow$ S = 5+(5 + 0.5) + (5 + 0.55) + (5 + 0.555) + ... to *n* terms

$\Rightarrow$ S = (5 + 5 + 5 + . . . to *n* terms) + {0.5 + 0.55 + 0.555 + ... to (*n* − 1)terms}

$\Rightarrow \quad S = 5n + 5\{0.1 + 0.11 + 0.111 + ... \text{ to } (n-1) \text{ terms}\}$

$\Rightarrow \quad S = 5n + \frac{5}{9}\{0.9 + 0.99 + 0.999 + ... \text{ to } (n-1)\text{terms}\}$

$$\Rightarrow \quad S = 5n + \frac{5}{9}\left\{\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right) + ... + \left(1-\frac{1}{10^{n-1}}\right)\right\}$$

$$\Rightarrow \quad S = 5n + \frac{5}{9}\left\{(n-1) - \left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3} + ... + \frac{1}{10^{n-1}}\right)\right\}$$

$$\Rightarrow \quad S = 5n + \frac{5}{9}\left[(n-1) - \frac{1}{10}\left\{\frac{1-\left(\frac{1}{10}\right)^{n-1}}{\left(1-\frac{1}{10}\right)}\right\}\right]$$

$$\Rightarrow \quad S = 5n + \frac{5}{9}\left\{(n-1) - \frac{1}{9}\left(1-\frac{1}{10^{n-1}}\right)\right\}$$

OR

Let a be the first term and r be the common ratio of the G.P. Then,

$$S = a + ar + ar^2 + ... + ar^{n-1} = a\left(\frac{r^n - 1}{r-1}\right)$$

$$P = a.\ ar\ ar^2 ...\ ar^{n-1}$$

$$= a^n r^{1+2+3+...+(n-1)} = a^n r^{\frac{n(n-1)}{2}} \text{ and,}$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + ... + \frac{1}{ar^{n-1}}$$

$$\Rightarrow \quad R = \frac{1}{a}\frac{\left\{\left(\frac{1}{r}\right)^n - 1\right\}}{\left\{\left(\frac{1}{r}\right) - 1\right\}} = \frac{1}{a}\left(\frac{1-r^n}{1-r}\right)\frac{1}{r^{n-1}}$$

$$\Rightarrow \quad R = \frac{1}{a}\left(\frac{r^n-1}{r-1}\right)\frac{1}{r^{n-1}}$$

$$\therefore \quad \frac{S}{R} = a\left(\frac{r^n-1}{r-1}\right).a\left(\frac{r-1}{r^n-1}\right)r^{n-1} = a^2 r^{n-1}$$

$$\Rightarrow \quad \left(\frac{S}{R}\right)^n = a^{2n} r^{n(n-1)} = \left\{a^n r^{\frac{n(n-1)}{2}}\right\}^{-2} = P^2$$

Hence, $\left(\frac{S}{R}\right)^n = P^2$

35. Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

But, now three coin is tossed, so the possible sample space contains,

S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Where S is sample space and here $n(S) = 8$

(A) 3 heads

Let us assume 'A' be the event of getting 3 heads

$$n(A) = 1$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(B) 2 heads

Let us assume 'B' be the event of getting 2 heads

$$n(B) = 3$$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}$$

(C) at least 2 heads

Let us assume 'C' be the event of getting at least 2 heads

$$n(C) = 4$$

$$\therefore \quad P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(D) at most 2 heads

Let us assume 'D' be the event of getting at most 2 heads

$$n(D) = 7$$

$$\therefore \quad P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$$

(E) no head

Let us assume 'E' be the event of getting no heads

$$n(E) = 1$$

$\therefore \quad P(E) = \frac{n(E)}{n(S)}$

$= \frac{1}{8}$

OR

Total number of letters in the word 'ASSASSINATION' are 13.

Out of which 3A's, 4S's, 2I's, 1T's and 1O.

(A) If four S's come consecutively in the word, then we considered these 4S's as 1 group. Now, the number of letters is 10.

S	S	S	S	A	A	A	I	I	N	N	T	O
1				9								

Number of words all S's are together

$= \frac{10!}{3!2!2!}$

Total number of words using letters of the word 'ASSASSINATION'

$= \frac{13!}{3!4!2!2!}$

$\therefore$ Required probability $= \frac{10!}{3!2!2!} \times \frac{3!4!2!2!}{13!}$

$= \frac{10! \times 4!}{13!} = \frac{4!}{13 \times 12 \times 11}$

$= \frac{24}{1716} = \frac{2}{143}$

(B) If 2's and 2 N's come together, then there as 10 alphabets.

Number of word when 2's and 2 N's are come together

$= \frac{10!}{3!4!} \times \frac{4!}{2!2!}$

$\therefore$ Required probability $= \frac{\frac{10!4!}{3!4!2!2!}}{\frac{13!}{3!4!2!2!}}$

$= \frac{4!10!}{2!2!3!4!} \times \frac{3!4!2!2!}{13!}$

$= \frac{4!10!}{13!} = \frac{4!}{13 \times 12 \times 11}$

$= \frac{24}{13 \times 12 \times 11} = \frac{2}{143}$

(C) If all A's are coming together, then there are 11 alphabets.

Number of words when all A's come together $= \frac{11!}{4!2!2!}$

Probability when all A's come together

$= \frac{\frac{11!}{4!2!2!}}{\frac{13!}{4!3!2!2!}} = \frac{11!}{4!2!2!} \times \frac{4!3!2!2!}{13!}$

$= \frac{11! \times 3!}{13!} = \frac{6}{13 \times 12} = \frac{1}{26}$

Required probability when all A's does not come together.

$= 1 - \frac{1}{26} = \frac{25}{26}$

(D) If no two A's are together, then first we arrange the alphabets except A's.

S	S	S	S	I	N	T	I	O	N

All the alphabets except A's are arranged in $\frac{10!}{4!2!2!}$.

There are 11 vacant places between these alphabets.

So, 3A's can be place in 11 places in ${}^{11}C_3$ ways $= \frac{11!}{3!8!}$

$\therefore$ Total number of words when no two A's together

$= \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$

Required probability $= \frac{11! \times 10!}{3!8!4!2!} \times \frac{4!3!2!2!}{13!}$

$= \frac{10!}{8! \times 13 \times 12}$

$= \frac{10 \times 9}{13 \times 12} = \frac{90}{156} = \frac{15}{26}$

SECTION - E

36. (A) Since, $(1 + i)z = (1 - i)\bar{z}$

$\Rightarrow \quad \frac{z}{\bar{z}} = \frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^2}{1-i^2}$

$= \frac{1+i^2-2i}{1+1} = -i$

$\Rightarrow \quad z = -i\,\bar{z}$

(B) $\because \overline{\bar{z}_1 \bar{z}_2} = \bar{z}_1 \cdot \bar{z}_2$

(C) Let

$$z = \frac{(2+i)x-i}{4+i} + \frac{(1-i)y+2i}{4i}$$

$$= \frac{2x+(x-1)i}{4+i} + \frac{y+(2-y)i}{4i} \times \frac{i}{i}$$

$$= \frac{(2x+(x-1)i)(4-i)}{(4+i)(4-i)} + \frac{-iy+(2-y)}{4}$$

$$= \frac{8x+x-1+i(4x-4-2x)}{16+1} + \frac{(2-y)-iy}{4}$$

$$= \frac{9x-1+i(2x-4)}{17} + \frac{2-y-iy}{4}$$

Since, z is real

$\Rightarrow \quad Im\, z = 0$

$\Rightarrow \quad \frac{2x-4}{17} - \frac{y}{4} = 0$

$\Rightarrow \quad 8x - 16 = 17y$

$\Rightarrow \quad 8x - 17y = 16$

OR

$$z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{(1+2i\sin\theta)}{(1+2i\sin\theta)}$$

$$= \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

$$= \frac{(3-4\sin^2\theta)+i(8\sin\theta)}{1+4\sin^2\theta}$$

Since, z is purely imaginary.

$\Rightarrow \quad Re(z) = 0$

$\Rightarrow \quad \frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow \quad \sin^2\theta = \frac{3}{4}$

$\Rightarrow \quad \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{3}\left(since, 0 < \theta \le \frac{\pi}{2}\right)$

37. (A) Let the cardinal numbers of sets A and B be $n(A)$ and $n(B)$ respectively.

Given, $n(A) + n(B) = 9$ ...(i)

Also, the cardinal number of the power set of $A = 2^{n(N)}$

and the cardinal number of the power set of $B = 2^{n(N)}$

Given, $\frac{2^{n(A)}}{2^{n(B)}} = \frac{8}{1} \Rightarrow 2^{n(A)-n(B)} = 2^3$

$\Rightarrow \quad n(A) - n(B) = 3$ ...(ii)

On adding (i) and (ii), we get

$2\,n(A) = 12$

$\Rightarrow \quad n(A) = 6$

Thus, the cardinal number of set A is 6.

(B) On subtracting (ii) from (i), we get

$2\,n(B) = 6 \Rightarrow n(B) = 3$

Thus, the cardinal number of set B is 3.

(C) We have,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

The value of $n(A \cup B)$ will be maximum when $n(A \cap B)$ will be minimum.

The minimum value of $n(A \cap B) = 0$.

So, maximum value of

$n(A \cap B) = n(A) + n(B) = 6 + 3 = 9$

OR

We have, $n(A \cup B) = n(A) + n(B) - n(A \cup B)$

The value of $n(A \cup B)$ will be minimum when $n(A \cap B)$ will be maximum.

The maximum value of $n(A \cap B) = 3$.

So, minimum value of

$n(A \cup B) = n(A) + n(B) = 6 + 3 - 3 = 6$

38. (A) The extremities of latus rectum are $(a, \pm 2a) = \left(\frac{1}{3}, \pm\frac{2}{3}\right)$

$\therefore$ Required distance

$$= \sqrt{\left(\frac{1}{3}-\frac{1}{3}\right)^2 + \left(\frac{2}{3}+\frac{2}{3}\right)^2}$$

$$= \frac{4}{3}$$

(B) The equation of directrix is

$$= \left(0 - \frac{1}{3}\right) = x - 0$$

$$\Rightarrow x = \frac{-1}{3}$$